CALCULATING THE RADIATIVE HEAT EXCHANGE BETWEEN TWO PARALLEL INFINITE METAL SURFACES

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UDC 536.3:535.34

A method is proposed to calculate the radiative heat exchange between two parallel infinite metal surfaces. The resulting heat flow – calculated exactly – does not deviate by more than 10% from that calculated according to an approximation formula.

We use the familiar Christiansen formula to calculate the radiative heat exchange between two gray plane-parallel surfaces:

$$q_{1-2} = \frac{\sigma (T_1^4 - T_2^4)}{\varepsilon_1^{-1} + \varepsilon_2^{-1} - 1}. \quad (1)$$

For metal surfaces exhibiting pronounced selectivity, formula (1) may yield an error of up to 200%.

Several authors [1, 2] attempted to make provision for selectivity.

As demonstrated in [3], the absorptivity a of a metallic surface at a temperature T_1 relative to the incident black radiation at a temperature T_2 may be regarded as equal to the emittance ε of this surface at a temperature $\overline{T} = \sqrt{T_1 T_2}$. The possible use and accuracy of the relationship

$$a = \varepsilon_{(\sqrt{T_1 T_2})} \quad (2)$$

is governed by the validity of the Drude formula for metals. Equation (2) was initially used by Hottel [1] for two identical metals, and then extended to any pair of metals. The Hottel formula has the form

$$q_H = \frac{\sigma (T_1^4 - T_2^4)}{\varepsilon_1^{-1} + \varepsilon_{\overline{T}}^{-1} - 1}. \quad (3)$$

It differs from (1) in that the emittance for the second surface was assumed equal to its absorptivity and equal to $\varepsilon_{\overline{T}}$. Formula (3) was more exact than formula (1); the calculational error which arises from the use of (3) reaches 25% [2].

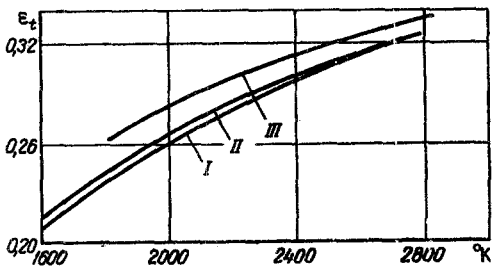

Fig. 1. Integral tungsten emittance: I) Worthing data; II) theoretical data based on the De Vos spectral characteristics; III) theoretical data based on the Riethof spectral characteristics.

If we proceed from the fact that because the absorptivities of the two metal surfaces are not equal to their emittances, although they remain constant on repeated reflection between the surfaces, the resulting formula [4, 5] has the form

$$q_{1-2} = \frac{\sigma (\varepsilon_1 a_1^{-1} T_1^4 - \varepsilon_2 a_2^{-1} T_2^4)}{a_1^{-1} + a_2^{-1} - 1}. \quad (4)$$

The quantities ε_1 and ε_2 in (4) are known; however, it remains unclear what values to use for a_1 and a_2; it is precisely for this reason that formula (4) has not yet found practical application in engineering calculations.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 16, No. 2, pp. 246-250, February, 1969. Original article submmited April 15, 1968.

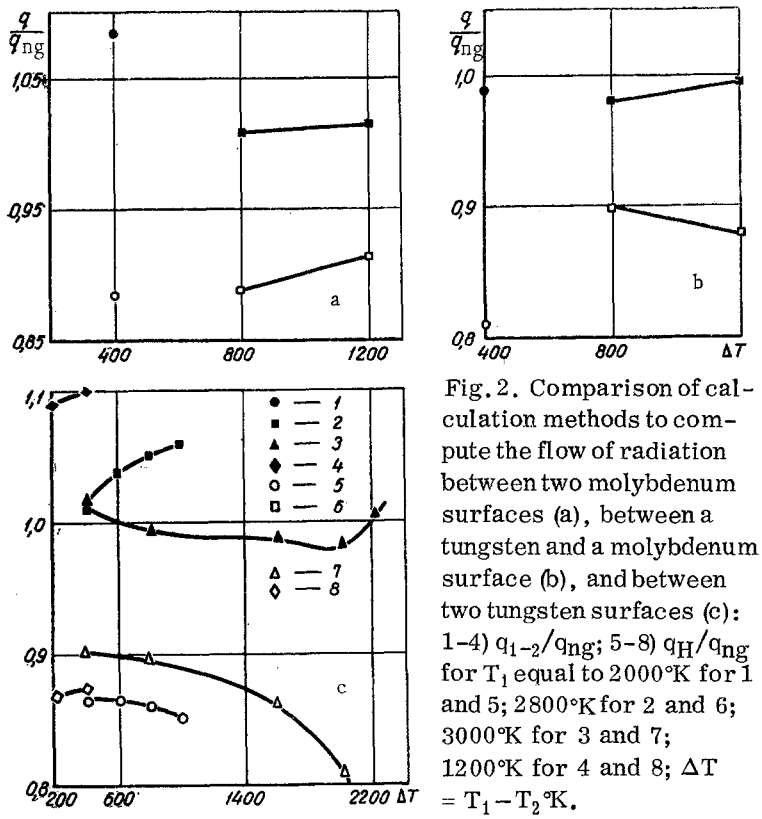

Fig. 2. Comparison of calculation methods to compute the flow of radiation between two molybdenum surfaces (a), between a tungsten and a molybdenum surface (b), and between two tungsten surfaces (c): 1-4) q_{1-2}/q_{ng}; 5-8) q_H/q_{ng} for T_1 equal to 2000°K for 1 and 5; 2800°K for 2 and 6; 3000°K for 3 and 7; 1200°K for 4 and 8; $\Delta T = T_1 - T_2$ °K.

Proceeding from the Hottel method (in which the value of $\varepsilon_{2(T_2)}$ has been replaced by $\varepsilon_{2(\overline{T})}$), in this paper the values of both a_1 and a_2 in (4) have been replaced by $\varepsilon_{1(\overline{T})}$ and $\varepsilon_{2(\overline{T})}$. As a result formula (4) assumes the following form:

$$q_{1-2} = \frac{\sigma\left[\varepsilon_1 \varepsilon^{-1}_{1(\overline{T})} T_1^4 - \varepsilon_2 \varepsilon^{-1}_{2(\overline{T})} T_2^4\right)}{\varepsilon^{-1}_{1(\overline{T})} + \varepsilon^{-1}_{2(\overline{T})} - 1}. \tag{5}$$

The validity of this approach can be verified by comparing the calculation of the resulting flow q_{1-2} calculated according to formula (5) with the calculation carried out in the assumption that the surfaces are diffuse nongray emitters (in the presence of spectral radiation characteristics) according to the formula

$$q_{ng} = \int_{\lambda=0}^{\lambda=\infty} \frac{d\lambda}{\varepsilon^{-1}_{\lambda,T_1} + \varepsilon^{-1}_{\lambda,T_2} - 1} (I_{\lambda,T_1} - I_{\lambda,T_2}). \tag{6}$$

A calculation such as this, for purposes of comparison with the Hottel formula, was performed in [2], and here the integral of (6) was written in finite differences (for computer calculation) and presented in the form

$$q_{ng} = \left[\sum_{\lambda=0.2}^{\lambda=20} \left(\frac{\Delta\lambda}{\varepsilon^{-1}_{\lambda,T_1} + \varepsilon^{-1}_{\lambda,T_2} - 1}\right)(I_{\lambda,T_1} - I_{\lambda,T_2})\right]$$
$$+ \left(\frac{1}{\varepsilon^{-1}_{20,T_1} + \varepsilon^{-1}_{20,T_2} - 1}\right)\left[\left(\sigma T_1^4 - \sum_{\lambda=0.2}^{\lambda=20} I_{\lambda,T_1}\Delta\lambda\right) - \left(\sigma T_2^4 - \sum_{\lambda=0.2}^{\lambda=20} I_{\lambda,T_2}\Delta\lambda\right)\right]. \tag{7}$$

We did the calculation on a Vega computer and we chose the following wavelength divisions:

1) 0.3 - 3 μm, $\Delta\lambda = 0.1$ μm;
2) 3 - 4 μm, $\Delta\lambda = 0.2$ μm;
3) 3 - 15 μm, $\Delta\lambda = 0.5$ μm.

With this division the calculation error is increased slightly; however, as demonstrated by our estimate, it does not exceed 1%.

TABLE 1. The Mo–Mo System

T_1, °K	T_2, °K	$\overline{T}$, °K	ε_1	ε_2	a_1 (a_2)	q_{ng}, W/cm^2	q_{1-2}, W/cm^2	q_H, W/cm^2
2800	2000	2360	0,262	0,212	0,242	47,71	48,11	42,49
2800	1600	2110	0,262	0,164	0,222	40,64	41,34	37,20
2000	1600	1790	0,212	0,164	0,189	6,712	7,261	5,963

TABLE 2. The W–Mo System

T_1, °K	T_2, °K	$\overline{T}$, °K	ε_1	ε_2	a_1	a_2	q_{ng}, W/cm^2	q_{1-2}, W/cm^2	q_H, W/cm^2
2800	2000	2360	0,334	0,212	0,314	0,240	46,13	45,45	41,39
2800	1600	2110	0,334	0,164	0,295	0,221	53,96	53,30	47,42
2000	1600	1790	0,283	0,164	0,261	0,188	8,188	8,084	6,590

TABLE 3. The W–W System

T_1, °K	T_2, °K	$\overline{T}$, °K	ε_1	ε_2	a_1 (a_2)	q_{1-2}, W/cm^2	q_H, W/cm^2	q_{ng}, W/cm^2
3000	2600	2790		0,312	0,322	43,40	38,61	42,90
	2200	2570		0,283	0,311	68,00	61,34	68,42
	1400	2050	0,334	0,182	0,269	86,21	75,70	87,40
	1000	1730		0,124	0,230	86,01	70,71	87,51
	900	1530		0,098	0,203	84,92	64,42	84,21
2000	1600	1780		0,215	0,238	8,965	7,520	8,680
	1400	1670		0,182	0,223	11,10	9,231	10,71
	1200	1530	0,262	0,152	0,203	12,13	9,922	11,52
	1000	1420		0,124	0,183	12,62	10,10	11,94
1200	1000	1090		0,124	0,134	0,536	0,436	0,501
	800	960	0,153	0,098	0,117	0,781	0,621	0,712

To work with (6) we need the spectral characteristics of the chosen metal pair over the entire wavelength range.

We calculated the q_{1-2} values for two metal pairs: molybdenum–molybdenum and tungsten–molybdenum; a test check was then conducted on the tungsten–tungsten pair, and for this we used the Branstaetter results [2]. The molybdenum and tungsten spectral characteristics were taken from Riethof [6] and extended to $\lambda > 4$ μm with the Drude formula. Since the ε_λ data were derived for normal radiation, they were converted according to the familiar relationship between hemispheric and normal emittance [7].

We should note that in comparing the magnitudes of the resulting flows q_{ng} and q_{1-2}, as calculated according to (5) and (6), the absolute accuracy of the employed spectral characteristics is of no significance. What is important in the calculation is the fact that the values of the integral hemispherical emittance and the values of $a_1 = \varepsilon_1(\sqrt{T_1T_2})$ and $a_2 = \varepsilon_2(\sqrt{T_1T_2})$ (for (4) and (5)) are calculated from the same spectral characteristics used in the calculation with (6).

For two identical metals we can simplify (5) to assume the following form:

$$q_{1-2} = \frac{\sigma}{2-\varepsilon}[\varepsilon_1 T_1^4 - \varepsilon_2 T_2^4], \tag{8}$$

where

$$\varepsilon_{1(\sqrt{T_1T_2})} = \varepsilon_{2(\sqrt{T_1T_2})} = \varepsilon.$$

The results of the calculation for molybdenum–molybdenum and tungsten–molybdenum systems are shown in Tables 1 and 2, respectively. We then calculated the tungsten–tungsten pair. Since the De Vos tungsten spectral characteristics were used in [2], we calculated the hemispherical tungsten emittance on the basis of the ε_λ values given in [8].

The results from the calculation of ε_t for tungsten and the comparison of these values with the Worthing [9] and Riethof [6] data are shown in Fig. 1. The calculation of the resulting flow q_{1-2} with the use of the above-cited ε_t values obtained from the De Vos spectral characteristics for ε_λ is summarized in Table 3.

To evaluate the errors associated with the replacement of the integral by a finite sum, we calculated the sum q_{1-2} at several characteristic points according to formula (7), with a division of $\Delta\lambda = 0.025$ μm. The error did not exceed 1%.

It is interesting to compare the resulting heat flows q_{ng} obtained through the use of the spectral characteristics from (6) with the flows q_H and q_{1-2} obtained from (3) and (5), respectively. The results of such a comparison for the Mo–Mo, W–Mo, and W–W systems are shown in Fig. 2a, b, c.

NOTATION

T	is the surface temperature, °K;
q	is the resulting radiative heat flow between two plane-parallel surfaces, W/cm^2;
a	is the surface absorptivity;
ε	is the surface emittance;
$\sigma = 5.67 \cdot 10^{-12}$	is the Stefan–Boltzmann constant, W/cm$^2 \cdot$ °K^4;
I	is the spectral blackbody radiation intensity, W/μm $\cdot$ cm^2;
λ	is the wavelength, μm;
q_H	is the heat flow calculated according to the Hottel formula.

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